Choosing and using representations to support instruction in number sense

Concrete and virtual materials can help students make sense of early number concepts by making important mathematical ideas visible. They can support students to connect **quantities**, **number words**, **written numerals**, **number order**, **magnitude**, **part-whole relationships**, and **place-value structure**.

This resource sits alongside the ENSSA subskill-specific recommendation sheets. The individual recommendation sheets explain which representations are useful for each skill area. This resource provides broader guidance on how to choose and use concrete and virtual materials carefully so they support mathematical understanding across early numeracy instruction.

The aim is not to use more materials, or to keep students using materials for longer than needed. The aim is to use materials deliberately, with clear teacher language, so students can move gradually from concrete materials to drawings, numerals, equations, and mental responses.

Why representations matter

The following concrete materials bring together key elements of **cardinality** (amount), **ordinality** (sequence), and **numerals** into integrated representations. These materials are especially useful because they help students see how quantity, order, and written number symbols are connected.

However, no single material is best for every task. The most useful material depends on the mathematical idea being taught. For example, a number track may support counting order, comparison, magnitude, and early operations, while a ten-frame may be better for seeing numbers in relation to 5 and 10. Teachers should choose the representation that makes the target idea visible.

A useful planning question is:

What mathematical idea is this material helping students see, say, or understand?

Start with the mathematical idea

Before choosing a material, identify the main mathematical idea students need to learn or practise.

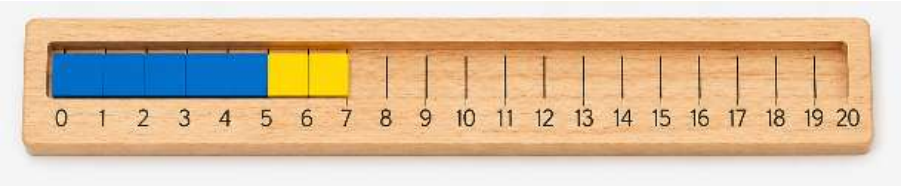
For example:

- If students need to know **how many**, use materials that support accurate counting, subitising, groupitising, or matching quantities to numerals.
- If students need to know **what comes before, after, or between**, use materials that make the number sequence visible.
- If students need to know **which number is more or less**, use materials that help them compare value.
- If students need to understand **how a whole number can be made from parts**, use materials that can be split, covered, moved, or recombined.
- If students need to understand **tens and ones**, use materials that clearly show groups of ten and extra ones.
- If students need to understand **number size, position, and distance**, use linear materials where larger numbers look, feel, or extend further than smaller numbers.

The same material can support different ideas depending on how it is used. Teacher language and task design determine what students are most likely to learn from the material.

Using number tracks well

Stern Number Tracks or other mathematically accurate number tracks



A mathematically accurate number track makes numerical magnitude and ordered number sequence visually clear. It can help students connect quantities, numerals, and number order, while also scaffolding towards a mental number line and later use of Base-10 materials.

Number tracks are a highly effective way of bringing together key components of early number sense, including quantity or cardinality, order or ordinality, and numerals. Integrating these components aligns with Science of Learning principles because it helps students connect related information in a clear and structured way.

A mathematically useful number track should include:

- a clear starting point
- proportional spacing that reflects equal numerical intervals
- numerals presented in the correct sequential order
- consistent left-to-right progression
- quantities, numerals, and number order presented together
- a clear visual representation of increasing magnitude as numbers increase

Number tracks can be purchased, but they can also be made simply using printed tracks, paper, or mini-whiteboards. For Foundation and Year 1 students, larger cubes, such as 2 cm cubes, are often easier to handle and align than very small cubes.

When using a number track, children need to be explicitly taught the conventions for how the representation works.

For example, teach students to:

- fill the number track with blocks from left to right
- use one block for each count
- avoid spaces between blocks
- avoid placing more than one block in the same position
- keep different coloured blocks grouped together when using colour to show part-whole relationships
- connect the final block, number word, and numeral to the total quantity

Number tracks can support several ENSSA recommendation areas. They are useful for counting and matching quantities to numerals, identifying before and after, comparing which number is more or less, building linear magnitude, and modelling simple addition or subtraction.

When the focus is **number sequence**, the number track helps students see the ordered count sequence. When the focus is **magnitude comparison**, the number track helps students explain which number comes before or after another number. When the focus is **magnitude estimation**, the track can help bridge from countable positions towards a more continuous understanding of number position and distance.

For further information, see:

Krasa, N., Tzanetopoulos, K., & Maas, C. (2023). *How Children Learn Math: The Science of Math Learning in Research and Practice*. Routledge.

Using hundreds charts well

Mathematically accurate hundreds charts

A mathematically accurate hundreds chart helps make the structure of the Base-10 number system visually clear. It can support number sequence, counting patterns, one more and one less, ten more and ten less, comparison within 100, and emerging place-value understanding.

A useful hundreds chart should make the structure of the number system easy to see. Key features include:

- numerals increasing by 1 across rows
- numerals increasing by 10 up columns*
- equal spacing between numerals
- clear decades, such as 10–19, 20–29, and 30–39
- repeating 0–9 patterns across rows
- numerals with the same ones digit aligned vertically, such as 5, 15, and 25

*This orientation is intentional: it aligns the hundreds chart with vertical representations such as thermometers and number lines, where values increase as you move upwards along the vertical axis.

90	91	92	93	94	95	96	97	98	99
80	81	82	83	84	85	86	87	88	89
70	71	72	73	74	75	76	77	78	79
60	61	62	63	64	65	66	67	68	69
50	51	52	53	54	55	56	57	58	59
40	41	42	43	44	45	46	47	48	49
30	31	32	33	34	35	36	37	38	39
20	21	22	23	24	25	26	27	28	29
10	11	12	13	14	15	16	17	18	19
0	1	2	3	4	5	6	7	8	9

When using a hundreds chart, students need explicit teaching to notice the structure. They may not automatically see rows, columns, decade patterns, or tens-and-ones relationships.

Useful teacher prompts include:

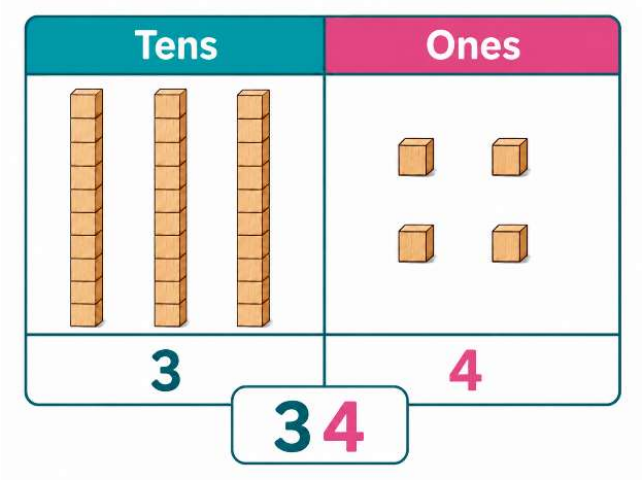
- “What changes as we move across the row?”
- “What changes as we move down the column?”
- “What do you notice about numbers in the same column?”

- "Where are the decade numbers?"
- "Which number is ten more?"
- "Which number is ten less?"

Hundreds charts are especially useful in Year 1 when students are extending number sequence and magnitude comparison to numbers within 100. They can also support students who are identifying missing numbers across decade transitions, such as 38, 39, __, 41.

Using Base-10 materials and simple place-value charts well

Base-10 blocks and simple place-value charts help Foundation and Year 1 students visually see that numbers are made from tens and ones. They support understanding of place value and help students compare numerical magnitude more efficiently.



Base-10 materials are particularly useful for showing:

- teen numbers as 10 and some more
- decade numbers as groups of ten
- two-digit numbers as tens and ones
- why 54 is more than 38
- why comparing tens comes before comparing ones in two-digit numbers
- how two-digit addition and subtraction can be represented

Students need explicit teaching that one ten represents a group of ten ones. They should have opportunities to build numbers, say the number, write the numeral, and explain the tens-and-ones structure.

For example:

- "14 is 10 and 4 more."
- "35 has 3 tens and 5 ones."
- "54 has 5 tens. 38 has 3 tens. Five tens is more than 3 tens, so 54 is more."
- "The tens are the same, so now we compare the ones."

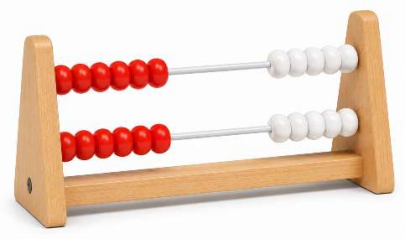
Simple place-value charts can be used alongside Base-10 materials to organise tens and ones. They should not replace concrete or visual understanding too early. Students usually need to build and explain numbers with materials before using a chart or numerals alone.

Other useful concrete materials

Many other concrete materials are commonly used in early numeracy. These materials are valuable when they are chosen for a clear instructional purpose and used with consistent mathematical language.

Useful concrete materials include:

- ten-frames
- five-frames
- connecting cubes or Unifix cubes
- counters
- bead strings
- rekenreks
- bundled paddle pop sticks
- dot cards
- number cards
- fingers
- part-part-whole mats



Ten-frames, five-frames, bead strings, and rekenreks can help students see numbers in relation to 5 and 10. They are useful for subitising, groupitising, comparing quantities, decomposing numbers, and developing make-ten strategies.

Connecting cubes are useful for counting, building quantities, decomposing numbers, comparing lengths, and developing linear magnitude. When they are used for magnitude work, cubes should touch and be arranged in a straight line so that length reflects quantity.

Counters, fingers, dot cards, and part-part-whole mats are useful for early counting, quantifying, subitising, decomposing, and modelling addition and subtraction. They should be used in ways that draw attention to number relationships, not only counting-by-ones.

Number cards are useful for matching numerals to quantities, ordering numbers, identifying missing numbers, and practising before and after language.

Bundled paddle pop sticks can support place-value understanding when students are learning that ten ones can be grouped as one ten. They can be especially useful when teachers want students to physically bundle and unbundle groups of ten.

Make the structure visible

Materials should be arranged so the important mathematical structure is easy to see.

For example:

- ten-frames should make five and ten visible
- bead strings and rekenreks should use colour changes intentionally, often in groups of 5 or 10
- connecting cubes used for magnitude should touch and be arranged in a straight line
- number tracks should be filled consistently from left to right
- Base-10 materials should clearly show the relationship between tens and ones
- hundreds charts should make decade patterns and tens-and-ones structure visible

When materials are arranged inconsistently, students may focus on surface features rather than the intended mathematical relationship.

Connect representations, then intentionally fade support

Students often benefit from seeing the same mathematical idea in more than one form at the same time. For example, a teacher might connect a quantity of counters, a ten-frame, the spoken number word, the written numeral, and a matching number sentence. This helps students understand that different representations can show the same number relationship.

Concrete materials are especially appropriate during the **acquisition stage of learning** (see *Using the Instructional Hierarchy to Guide ENSSA Recommendations*), when students are first learning what a concept or procedure means and how to respond accurately. At this stage, materials help make the mathematics visible and support teacher modelling, guided practice, feedback and correction.

As students become accurate and independent, concrete and visual supports should be intentionally faded. This does not mean removing representations altogether. It means reducing reliance on them so students can respond more efficiently using drawings, numerals, equations, oral explanations and mental strategies.

Concrete materials, drawings, diagrams, numerals, equations, and oral explanations should not be treated as separate stages that always happen one after the other. Instead, use them together when this helps students make connections, then intentionally fade the concrete and visual supports as students become more accurate and independent.

For example, students might first solve a comparison using two quantities and numerals together. Next, the teacher might show only the numerals but keep a number track available for checking. Later, students compare the numerals directly and explain their thinking without needing the track.

Similarly, when teaching addition and subtraction, students might connect counters or ten-frames with drawings and number sentences. As students become accurate, the teacher reduces reliance on materials and moves towards numeral-only practice, oral responses, and mental strategies.

The aim is for students to understand the relationship well enough to use numerals and mental responses efficiently, while still being able to return to a representation when they need to explain, check, or correct their thinking.

Using virtual materials carefully

Virtual manipulatives can support early numeracy when they preserve the mathematical structure and allow students to act, explain, check, or compare.

Useful virtual materials may include:

- virtual Base-10 blocks
- virtual place-value charts
- digital hundreds charts
- interactive number lines

- virtual rekenreks
- mathematics apps or digital games involving:
  - comparing numerals
  - ordering numbers
  - identifying greater than or less than
  - comparing quantities paired with numerals
  - placing numbers on number lines

Virtual materials can be useful for whole-class modelling, quick practice, independent practice, and checking understanding. They can also provide access to materials that may not be physically available.

However, digital activities are not automatically effective. Choose virtual tools that make the mathematical idea clear. Be cautious with activities that rely mainly on speed, guessing, animation, or reward features before students are accurate.

Useful questions to ask when choosing a virtual tool include:

- Does it show the mathematical structure clearly?
- Can students explain what they are doing?
- Does it connect quantities, numerals, order, magnitude, or place value?
- Does it allow correction and feedback?
- Does it support the instructional goal, or is it mainly a game?

Common pitfalls to avoid

Avoid using materials in ways that hide the intended mathematics.

Common pitfalls include:

- using materials without clear teacher language
- asking students to move objects without explaining the number relationship
- using unstructured collections when a structured representation would better support the goal
- fading concrete supports before students understand the underlying relationship
- keeping students on concrete materials long after they are ready for drawings, numerals, or mental responses
- using virtual games that reward speed or guessing before students are accurate
- arranging materials inconsistently across lessons
- using colour changes, gaps, or spacing in ways that distract from the intended number relationship

The key question is always:

What mathematical idea is this material helping students see, say, or understand?

How this connects to the ENSSA recommendation sheets

The ENSSA recommendation sheets provide subskill-specific guidance. Use those sheets to identify the instructional goal, recommended routines, vocabulary, representations, stem sentences, and review activities.

This resource supports those sheets by explaining broader principles for using materials well across early numeracy instruction.

It is especially relevant when teachers are:

- choosing materials for a lesson or intervention group
- deciding whether students need concrete, visual, numeral, or mental practice
- preparing routines that use number tracks, ten-frames, hundreds charts, Base-10 materials, or virtual tools
- checking that representations are being used consistently across classrooms
- planning how to move students from materials towards more efficient and independent responses

The subskill recommendation sheets should remain the main guide for what to teach, and in many cases, make direct recommendations about which representations best support which skills. This resource helps teachers make thoughtful choices about how to represent mathematics more broadly, across the maths curriculum.